Wahlaufgaben

Aufgabe 2006 W1a:

In der Figur ABCDE sind gegeben:

$\overline{AB} = 12,2\,cm$

$\overline{AE} = 8,5\,cm$

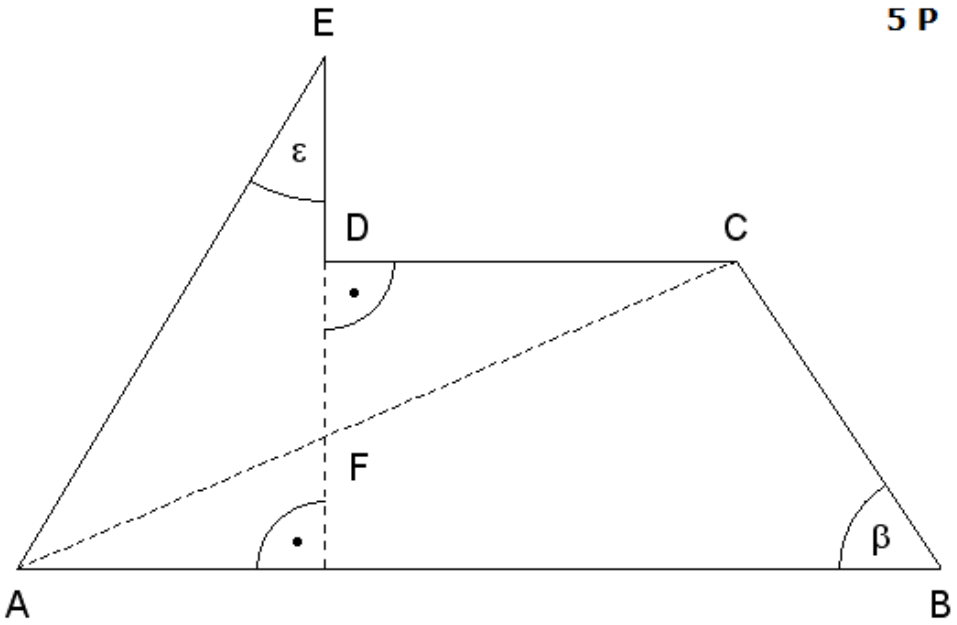
$\overline{BC} = 4,7\,cm$

$\beta = 59,0°$

$\varepsilon = 41,0°$

Berechnen Sie die Länge $\overline{DF}$.

5 P



Strategie 2006 W1a:

Gegeben:

$\overline{AB} = 12,2\,cm$

$\overline{AE} = 8,5\,cm$

$\overline{BC} = 4,7\,cm$

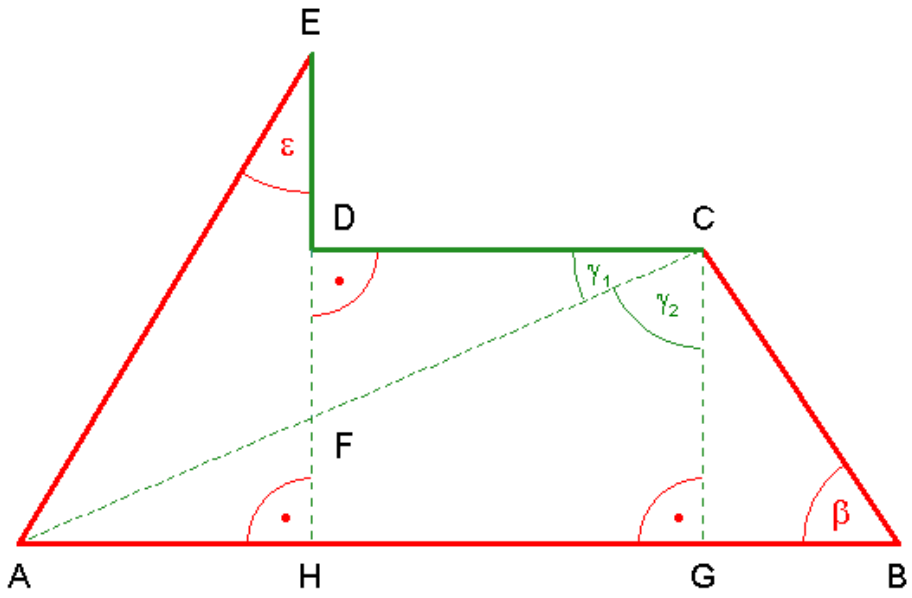
$\beta = 59,0°$

$\varepsilon = 41,0°$

Gesucht:

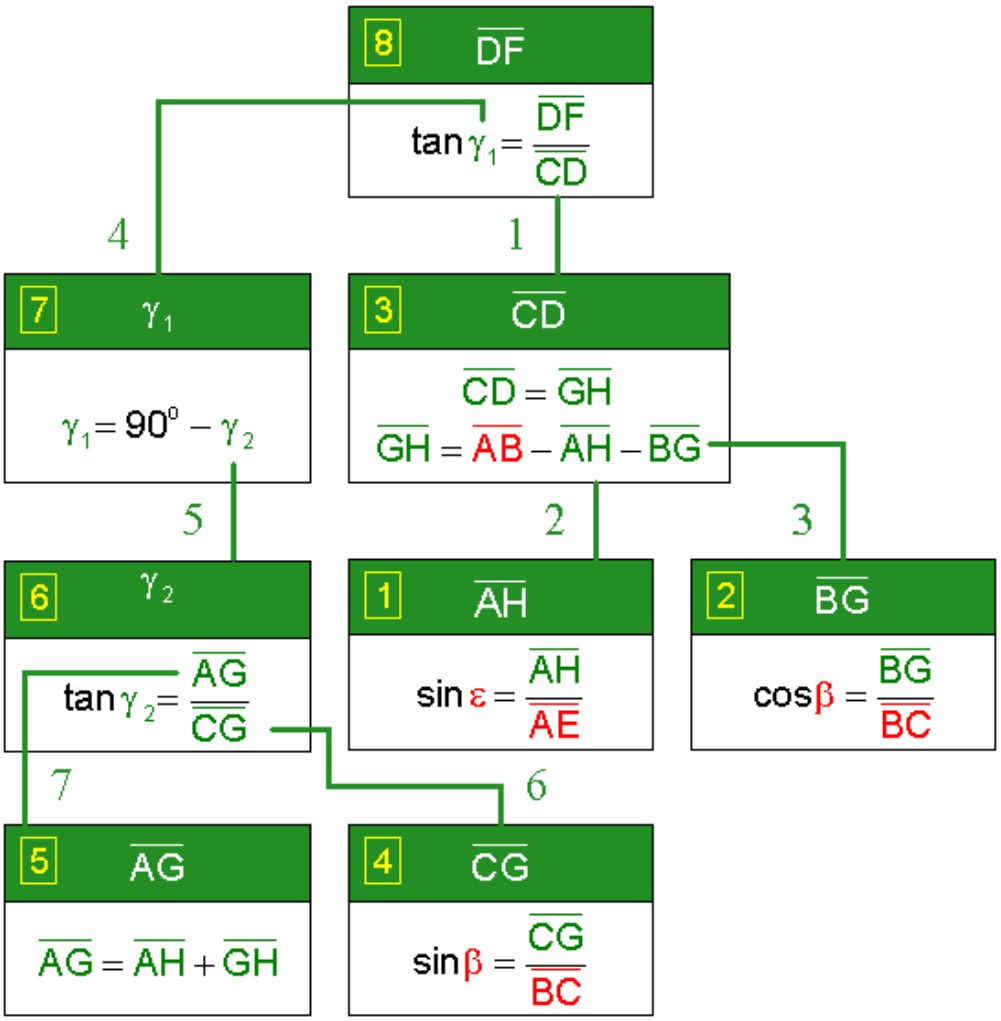
$\overline{DF}$

Skizze:



Strategie 2006 W1a:

Struktogramm:



Lösung 2006 W1a:

1. Berechnung der Strecke $\overline{AH}$:

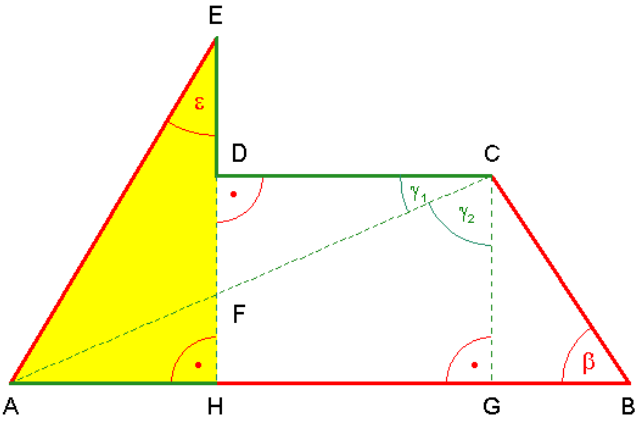
$$\sin\varepsilon = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{AH}}{\overline{AE}}$$ Sinusfunktion im rechtwinkligen gelben Teildreieck

$$\sin 41^\circ = \frac{\overline{AH}}{8,5}$$

$$0,6561 = \frac{\overline{AH}}{8,5}$$ Seiten tauschen

$$\frac{\overline{AH}}{8,5} = 0,6561 \quad | \cdot 8,5$$

$$\overline{AH} = 5,58\,\text{cm}$$



Lösung 2006 W1a:

2. Berechnung der Strecke $\overline{BG}$:

$$\cos\beta = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BG}}{\overline{BC}}$$

Kosinusfunktion im rechtwinkligen hellblauen Teildreieck

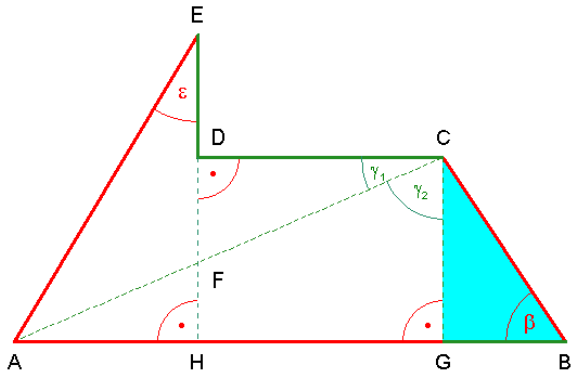
$$\cos 59^\circ = \frac{\overline{BG}}{4,7}$$

$$0,5150 = \frac{\overline{BG}}{4,7}$$

Seiten tauschen

$$\frac{\overline{BG}}{4,7} = 0,5150 \qquad |\cdot 4,7$$

$$\underline{\overline{BG} = 2,42\,\text{cm}}$$

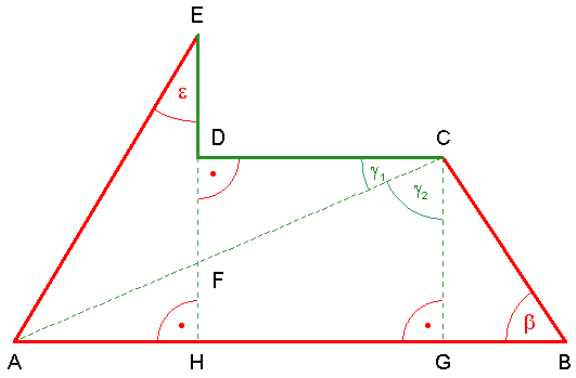


3. Berechnung der Strecke $\overline{CD}$:

$$\overline{CD} = \overline{GH} = \overline{AB} - \overline{AH} - \overline{BG}$$

$$\overline{CD} = \overline{GH} = 12,2 - 5,58 - 2,42$$

$$\underline{\overline{CD} = \overline{GH} = 4,2\,\text{cm}}$$



4. Berechnung der Strecke $\overline{CG}$:

$$\sin\beta = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{CG}}{\overline{BC}}$$

Sinusfunktion im rechtwinkligen hellblauen Teildreieck

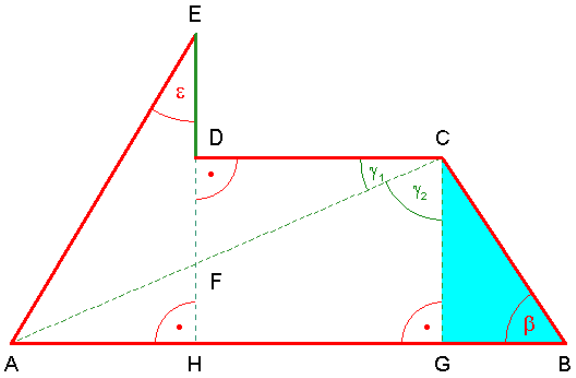
$$\sin 59^\circ = \frac{\overline{CG}}{4,7}$$

$$0,8572 = \frac{\overline{CG}}{4,7}$$

Seiten tauschen

$$\frac{\overline{CG}}{4,7} = 0,8572 \qquad |\cdot 4,7$$

$$\underline{\overline{CG} = 4,03\,\text{cm}}$$



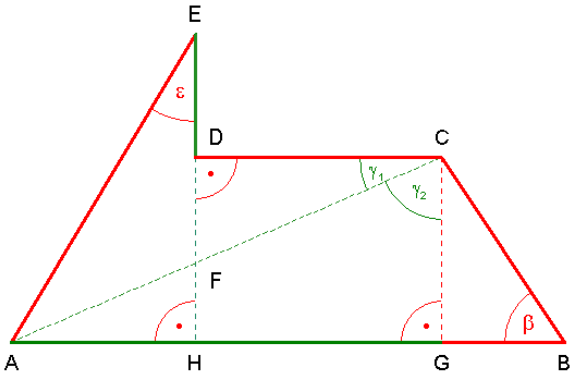
Lösung 2006 W1a:

5. Berechnung der Strecke $\overline{AG}$:

$$\overline{AG} = \overline{AH} + \overline{GH}$$

$$\overline{AG} = 5,58 + 4,2$$

$$\underline{\overline{AG} = 9,78\,cm}$$



6. Berechnung des Winkels γ_2:

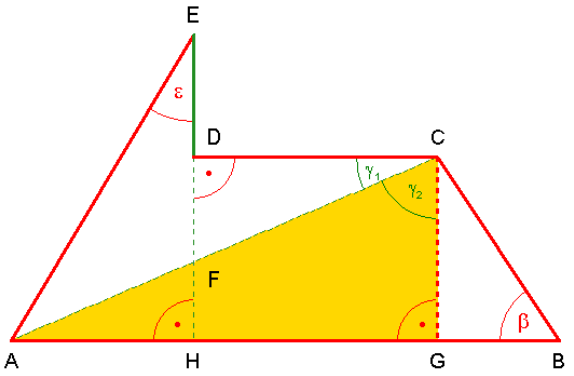
$$\tan\gamma_2 = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{AG}}{\overline{CG}}$$

Tangensfunktion im rechtwinkligen orangefarbigen Teildreieck

$$\tan\gamma_2 = \frac{9,78}{4,03}$$

$$\tan\gamma_2 = 2,4268$$

$$\underline{\gamma_2 = 67,6°}$$

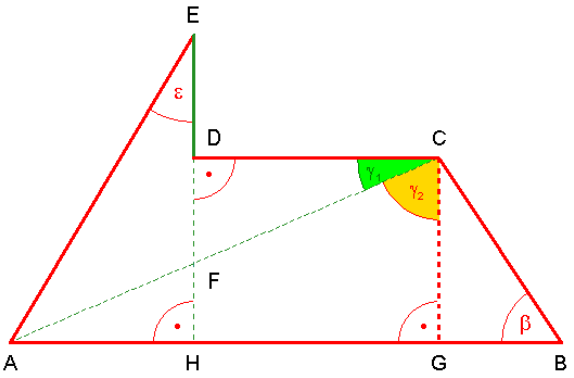


7. Berechnung des Winkels γ_1:

$$\gamma_1 = 90° - \gamma_2$$

$$\gamma_1 = 90° - 67,6°$$

$$\underline{\gamma_1 = 22,4°}$$



8. Berechnung der Strecke $\overline{DF}$:

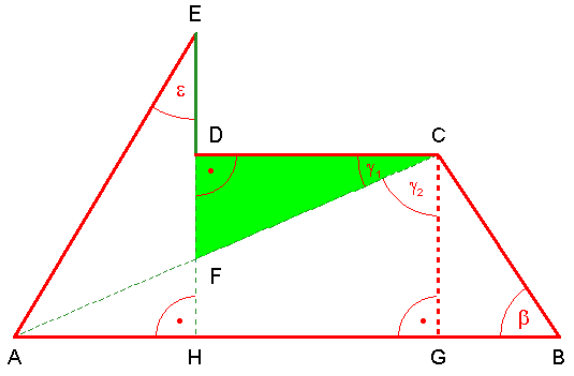
$$\tan\gamma_1 = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{DF}}{\overline{CD}}$$

Tangensfunktion im rechtwinkligen grünen Teildreieck

$$\tan 22,4° = \frac{\overline{DF}}{4,2}$$

$$0,4122 = \frac{\overline{DF}}{4,2}$$

Seiten tauschen

$$\frac{\overline{DF}}{4,2} = 0,4122 \qquad \cdot 4,2$$

$$\underline{\underline{\overline{DF} = 1,73\,cm}}$$