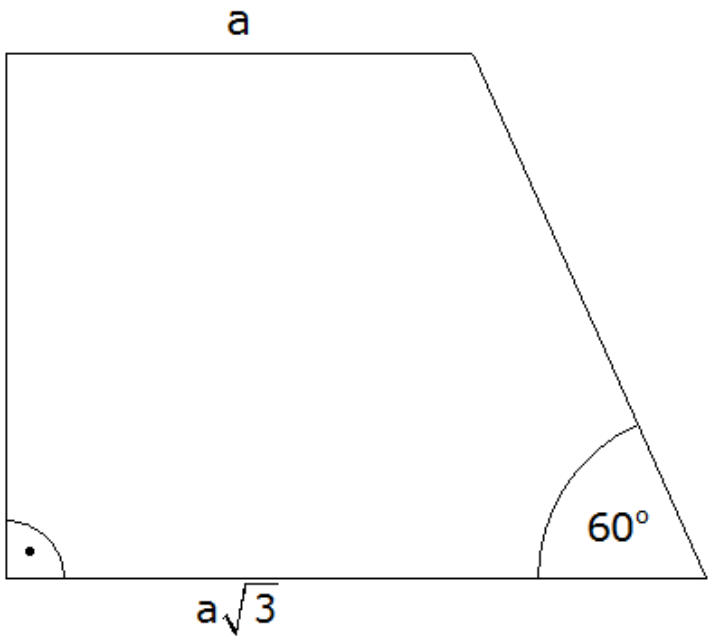
Aufgabe 1995 4c: **3 P**

Zeigen Sie ohne Verwendung gerundeter Werte, daß der Flächeninhalt des nebenstehenden Trapezes mit der Formel

$$A = a^2\sqrt{3}$$

berechnet werden kann.



Strategie 1995 4c:

Gegeben:

$\overline{AB} = a\sqrt{3}$

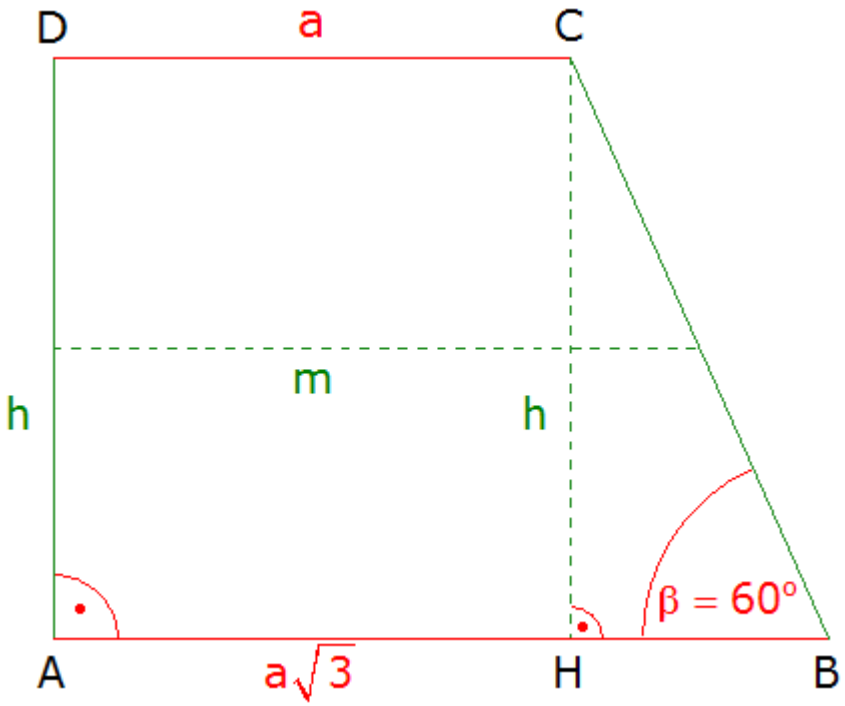
$\overline{CD} = a$

$\beta = 60°$

Gesucht:

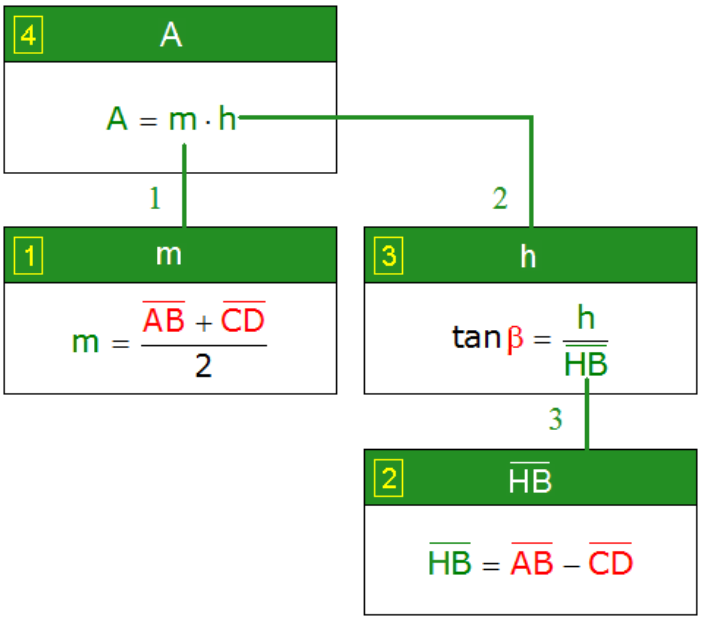
$A = a^2\sqrt{3}$

Skizze:



Strategie 1995 4c:

Struktogramm:



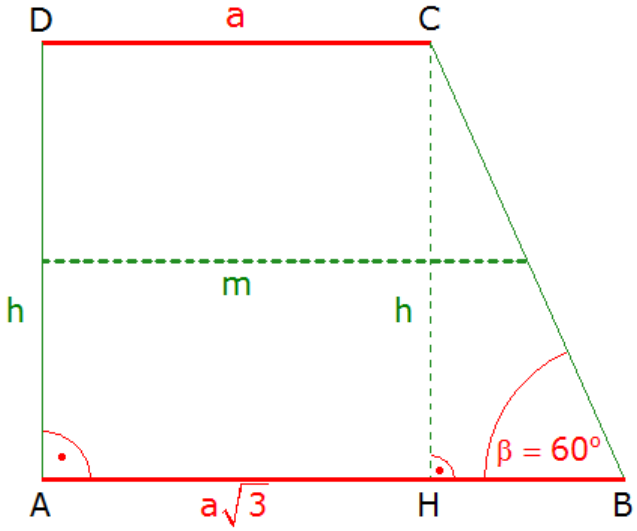
Lösung 1995 4c:

1. Berechnung der Trapezmittellinie m:

$$m = \frac{\overline{AB} + \overline{CD}}{2}$$

$$m = \frac{a\sqrt{3} + a}{2}$$

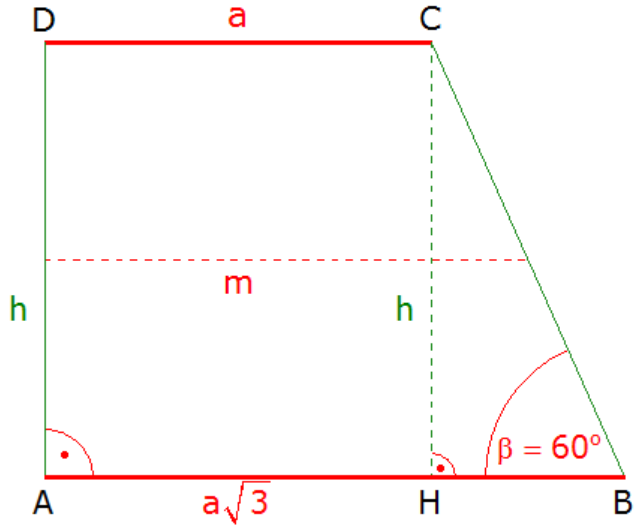
$$m = \frac{a\sqrt{3}}{2} + \frac{a}{2}$$



2. Berechnung der Strecke HB:

$$\overline{HB} = \overline{AB} - \overline{CD}$$

$$\overline{HB} = a\sqrt{3} - a$$



Lösung 1995 4c:

3. Berechnung der Trapezhöhe h:

$$\tan\beta = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{h}{\overline{HB}}$$ Tangensfunktion im rechtwinkligen gelben Teildreieck

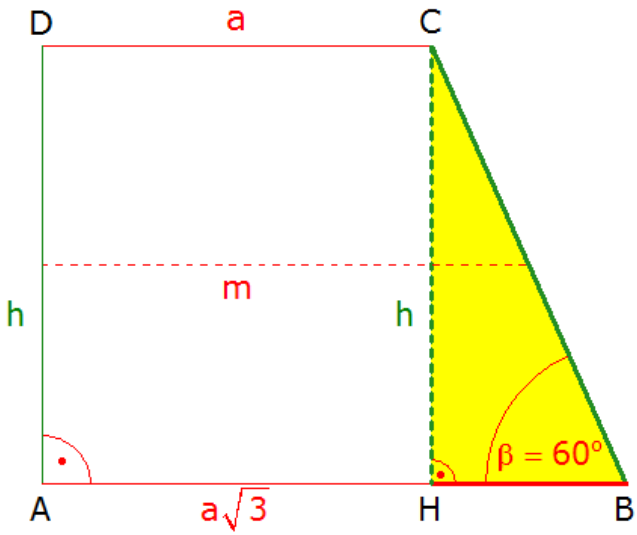
$$\tan 60^\circ = \frac{h}{a\sqrt{3}-a}$$ $\tan 60^\circ = \sqrt{3}$

$$\sqrt{3} = \frac{h}{a\sqrt{3}-a}$$ Seiten tauschen

$$\frac{h}{a\sqrt{3}-a} = \sqrt{3} \quad \left|\cdot\left(a\sqrt{3}-a\right)\right.$$

$$h = \sqrt{3}\cdot\left(a\sqrt{3}-a\right)$$

$$\underline{h = 3a - \sqrt{3}\cdot a}$$



4. Berechnung der Trapezfläche A:

$$A = m\cdot h$$

$$A = \left(\frac{a\sqrt{3}}{2} + \frac{a}{2}\right)\cdot\left(3a - \sqrt{3}\cdot a\right)$$

$$A = \left(\frac{a\sqrt{3}}{2} + \frac{a}{2}\right)\cdot\left(3a - \sqrt{3}\cdot a\right)$$ Summe mal Summe

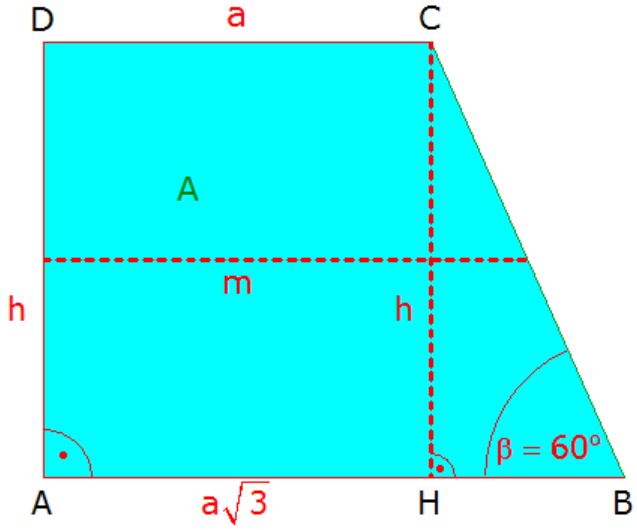
$$A = \frac{a\sqrt{3}}{2}\cdot 3a - \frac{a\sqrt{3}}{2}\cdot\sqrt{3}\cdot a + \frac{a}{2}\cdot 3a - \frac{a}{2}\cdot\sqrt{3}\cdot a$$

$$A = 1{,}5\sqrt{3}a^2 - 1{,}5a^2 + 1{,}5a^2 - 0{,}5\sqrt{3}a^2$$

$$A = 1{,}5\sqrt{3}a^2 - \cancel{1{,}5a^2 + 1{,}5a^2} - 0{,}5\sqrt{3}a^2$$

$$A = 1{,}5\sqrt{3}a^2 - 0{,}5\sqrt{3}a^2$$

$$A = 1{,}0\sqrt{3}a^2$$

$$A = \sqrt{3}a^2$$

$$\underline{\underline{A = a^2\sqrt{3}}}$$