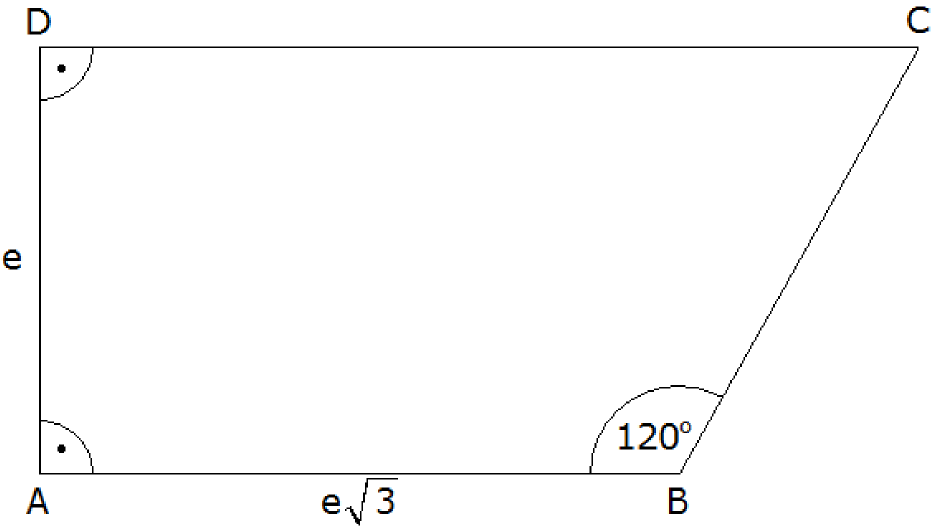
Aufgabe 1994 3c: **3 P**

Zeigen Sie ohne Verwendung gerundeter Werte, daß sich Umfang und Flächeninhalt des Trapezes in Abhängigkeit von e mit den Formeln

$$u = e\left(1 + 3\sqrt{3}\right)$$

$$A = \frac{7}{6}e^2\sqrt{3}$$

berechnen lassen.



Strategie 1994 3c:

Gegeben:

Trapez

$\overline{AB} = e\sqrt{3}$

$\overline{AD} = e$

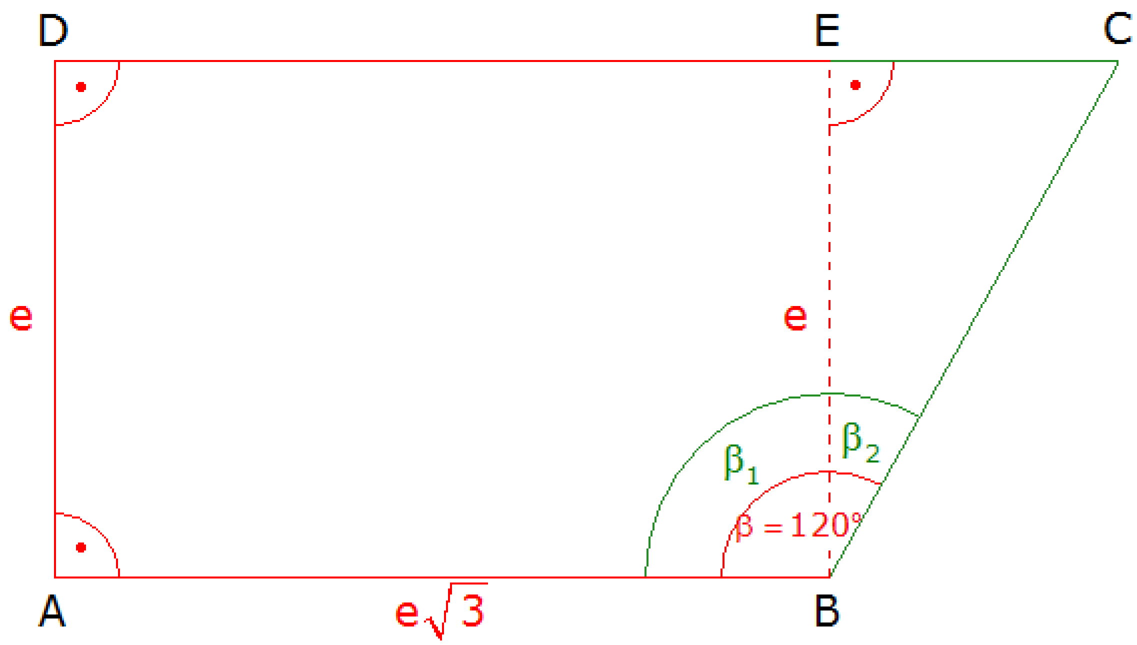
$\beta = 120°$

Gesucht:

$u = e\left(1 + 3\sqrt{3}\right)$

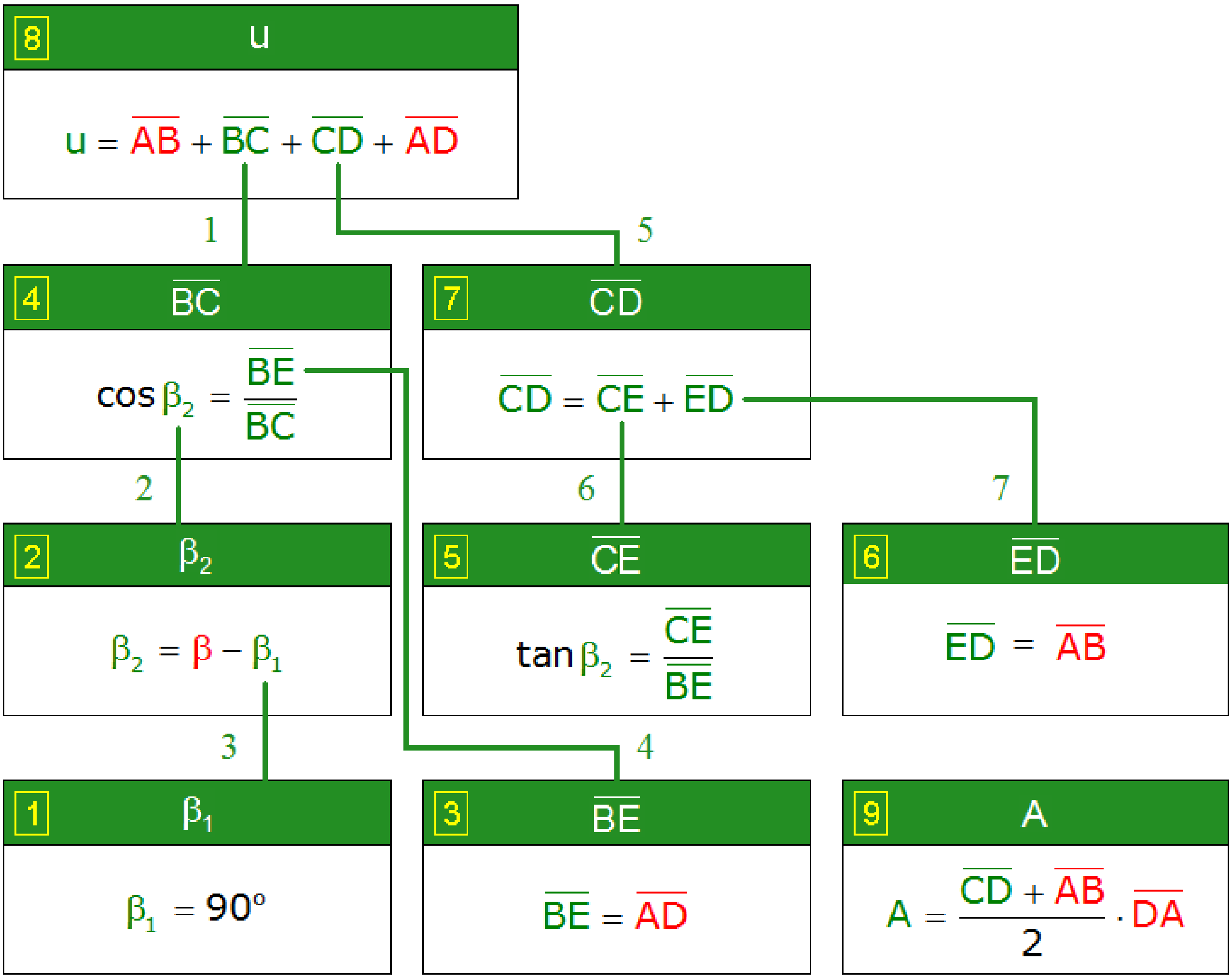
$A = \frac{7}{6}e^2\sqrt{3}$

Skizze:



Strategie 1994 3c:

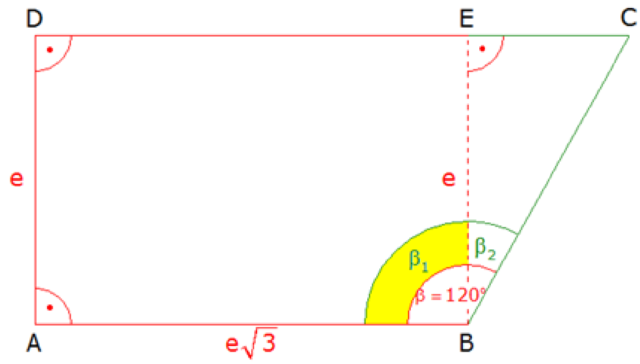
Struktogramm:



Lösung 1994 3c:

1. Bestimmung des Winkels β_1:

$\beta_1 = 90^\circ$

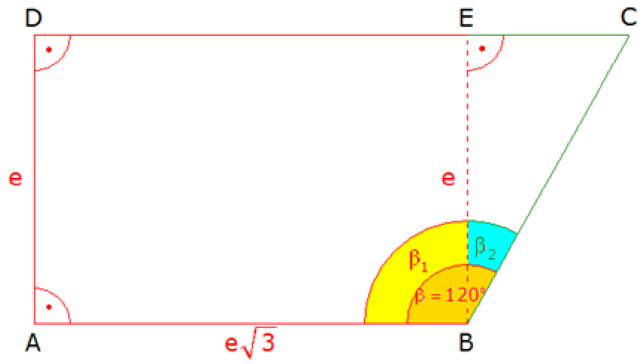


2. Berechnung des Winkels β_2:

$\beta_2 = \beta - \beta_1$

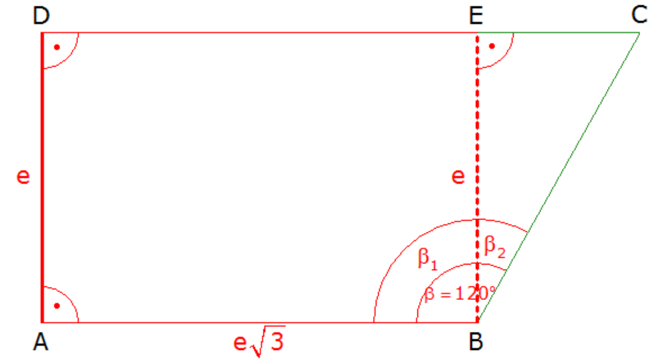
$\beta_2 = 120^\circ - 90^\circ$

$\beta_2 = 30^\circ$



Lösung 1994 3c:

3. Berechnung der Strecke $\overline{BE}$:



$$\overline{BE} = \overline{AD}$$

$$\overline{BE} = e$$

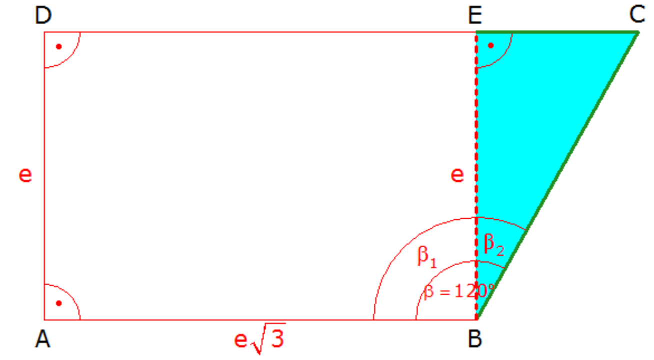
4. Berechnung der Strecke $\overline{BC}$:

$$\cos\beta_2 = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BE}}{\overline{BC}}$$

Kosinusfunktion im rechtwinkligen hellblauen Teildreieck BCE

$$\cos 30° = \frac{e}{\overline{BC}}$$

$$\cos 30° = \frac{1}{2}\sqrt{3}$$



$$\frac{1}{2}\sqrt{3} = \frac{e}{\overline{BC}} \qquad |\cdot \overline{BC}$$

$$\overline{BC} \cdot \frac{1}{2}\sqrt{3} = e \qquad |\cdot 2$$

$$\overline{BC} \cdot \sqrt{3} = 2e \qquad |:\sqrt{3}$$

$$\overline{BC} = \frac{2e}{\sqrt{3}}$$

$$\overline{BC} = \frac{2e\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

Nenner rational machen

$$\overline{BC} = \frac{2e\cdot\sqrt{3}}{3}$$

$$\underline{\overline{BC} = \frac{2}{3}e\sqrt{3}}$$

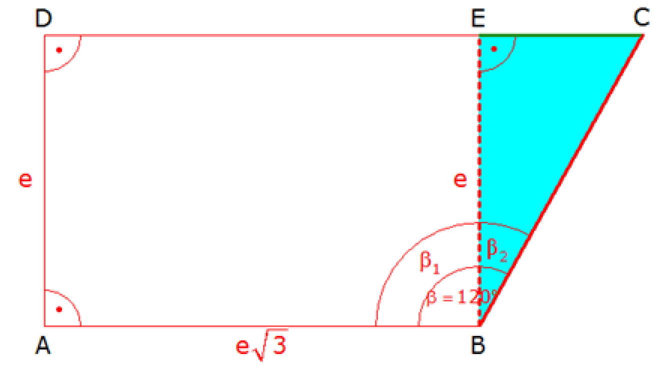
5. Berechnung der Strecke $\overline{CE}$:

$$\tan\beta_2 = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{CE}}{\overline{BE}}$$

Tangensfunktion im rechtwinkligen hellblauen Teildreieck BCE

$$\tan 30° = \frac{\overline{CE}}{e}$$

$$\tan 30° = \frac{1}{3}\sqrt{3}$$



$$\frac{1}{3}\sqrt{3} = \frac{\overline{CE}}{e}$$

Seiten tauschen

$$\frac{\overline{CE}}{e} = \frac{1}{3}\sqrt{3} \qquad |\cdot e$$

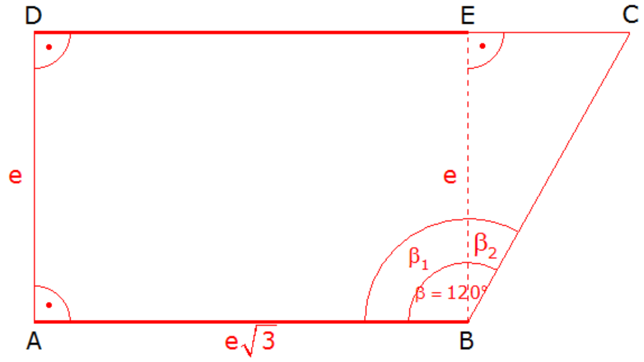
$$\underline{\overline{CE} = \frac{1}{3}\cdot e\cdot\sqrt{3}}$$

Lösung 1994 3c:

6. Berechnung der Strecke $\overline{ED}$:

$$\overline{ED} = \overline{AB}$$

$$\overline{ED} = e\sqrt{3}$$



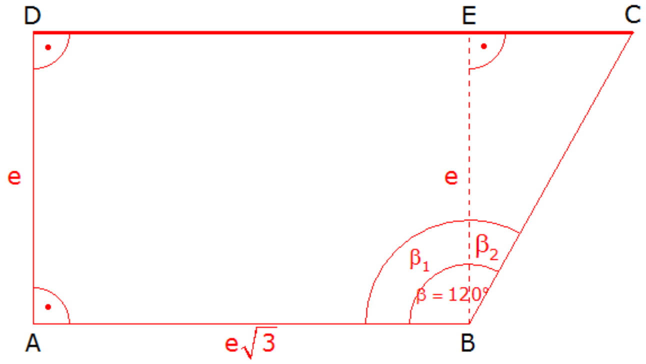
7. Berechnung der Strecke $\overline{CD}$:

$$\overline{CD} = \overline{CE} + \overline{ED}$$

$$\overline{CD} = \frac{1}{3}e\sqrt{3} + e\sqrt{3}$$

$$\overline{CD} = \frac{1}{3}e\sqrt{3} + \frac{3}{3}e\sqrt{3}$$

$$\overline{CD} = \frac{4}{3}e\sqrt{3}$$



8. Berechnung des Trapezumfangs u:

$$u = \overline{AB} + \overline{BC} + \overline{CD} + \overline{AD}$$

$$u = e\sqrt{3} + \frac{2}{3}e\sqrt{3} + \frac{4}{3}e\sqrt{3} + e$$

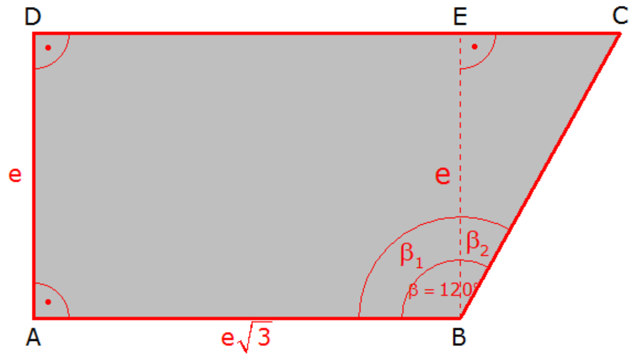
$$u = \frac{3}{3}e\sqrt{3} + \frac{2}{3}e\sqrt{3} + \frac{4}{3}e\sqrt{3} + e$$

$$u = \frac{9}{3}e\sqrt{3} + e$$

$$u = 3e\sqrt{3} + e$$

$$u = e + 3e\sqrt{3}$$

$$u = e\left(1 + 3\sqrt{3}\right)$$



9. Berechnung der Trapezfläche A:

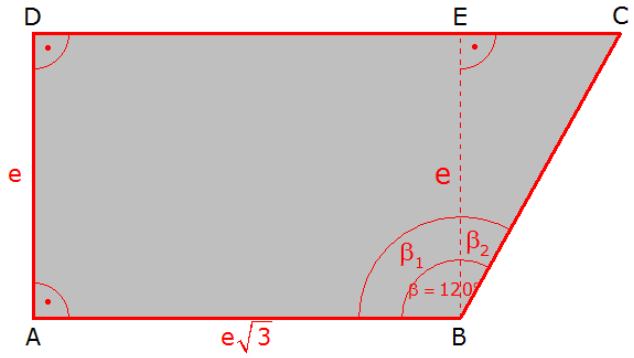
$$A = m \cdot \overline{AD}$$ Flächenformel Trapez

$$A = \frac{\overline{CD} + \overline{AB}}{2} \cdot \overline{AD}$$

$$A = \frac{\frac{4}{3}e\sqrt{3} + e\sqrt{3}}{2} \cdot e$$

$$A = \frac{\frac{4}{3}e\sqrt{3} + \frac{3}{3}e\sqrt{3}}{2} \cdot e$$

$$A = \frac{\frac{7}{3}e\sqrt{3}}{2} \cdot e$$



Lösung 1994 3c:

$$A = \frac{7}{3} \cdot \frac{1}{2} e \cdot e\sqrt{3}$$

$$\underline{\underline{A = \frac{7}{6} e^2 \sqrt{3}}}$$