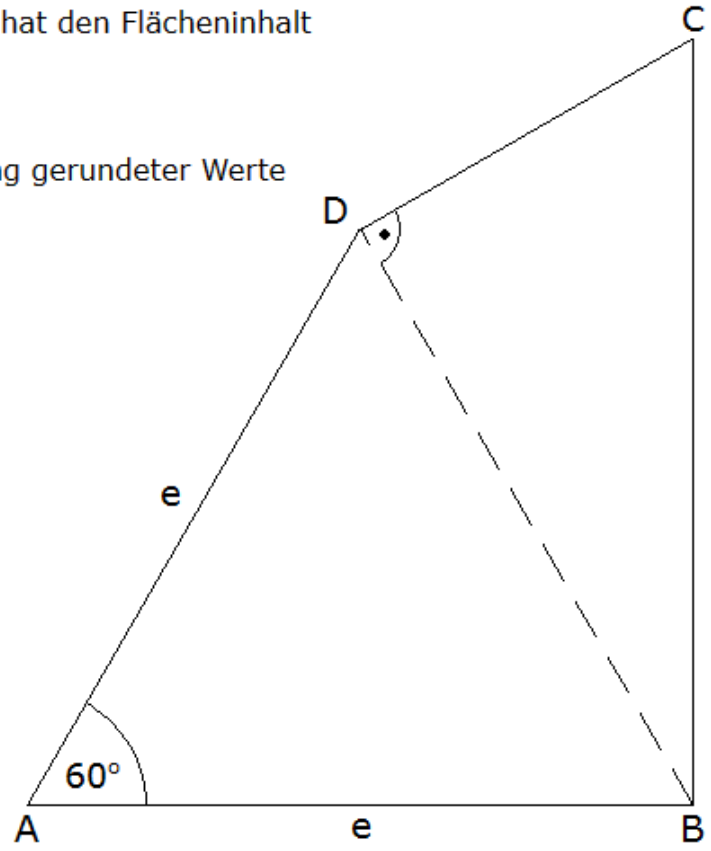
Aufgabe 1993 3c: **3 P**

Nebenstehendes Viereck ABCD hat den Flächeninhalt

$$A = \frac{5e^2}{12}\sqrt{3}$$

Berechnen Sie ohne Verwendung gerundeter Werte den Winkel $CBD = \varphi$.



Strategie 1993 3c:

Gegeben:

$A = \frac{5e^2}{12}\sqrt{3}$

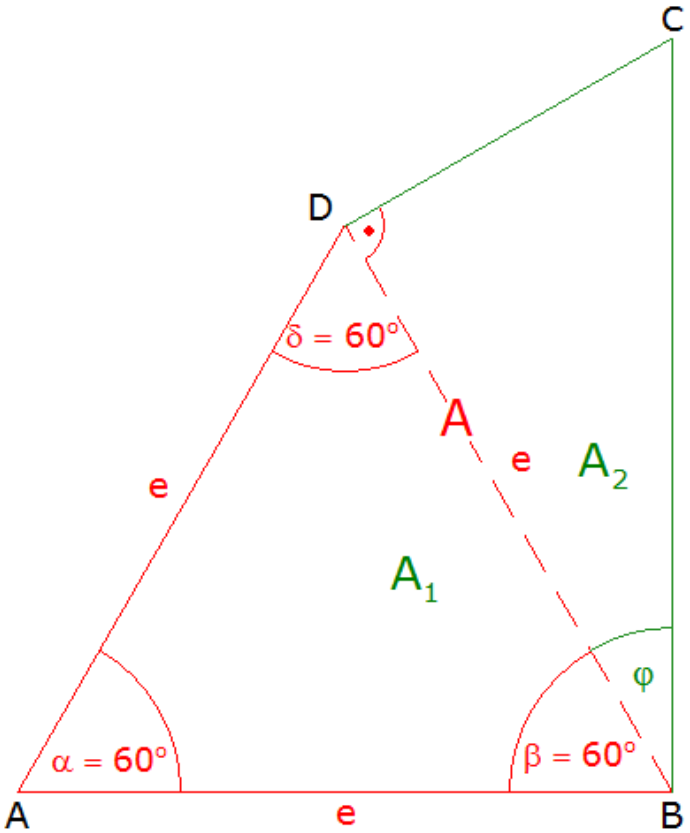
$\alpha = \beta = \delta = 60^\circ$

$\overline{BD} = e$

Gesucht:

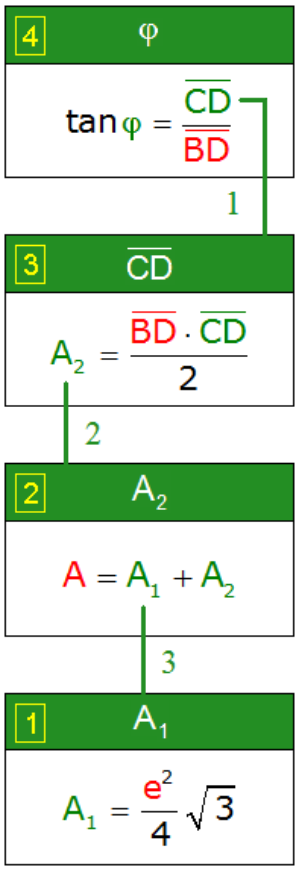
$\sphericalangle CBD = \varphi$

Skizze:



Strategie 1993 3c:

Struktogramm:

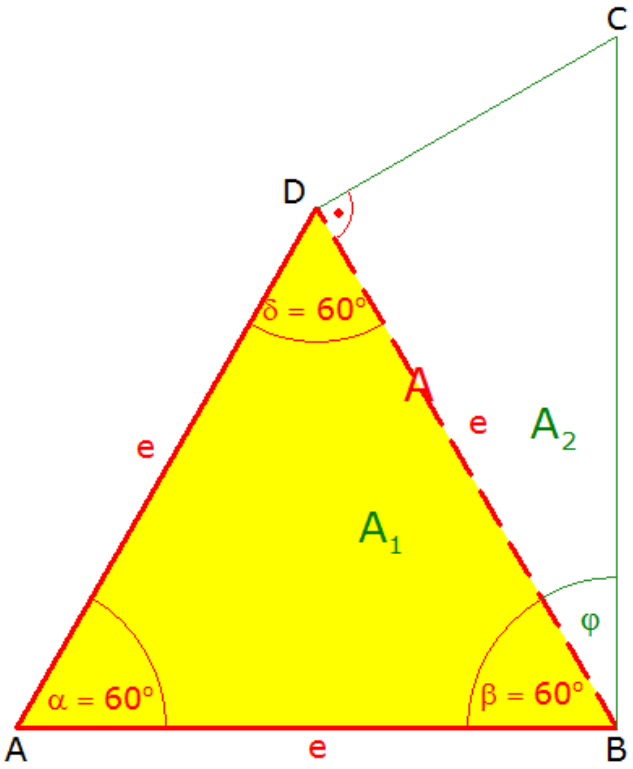


Lösung 1993 3c:

1. Berechnung der Dreiecksfläche A_1:

$$A_1 = \frac{\overline{AB}^2}{4}\sqrt{3}$$ Flächenformel gleichseitiges Dreieck

$$A_1 = \frac{e^2}{4}\sqrt{3}$$



Lösung 1993 3c:

2. Berechnung der Dreiecksfläche A_2:

$$A = A_1 + A_2$$

$$\frac{5e^2}{12}\sqrt{3} = \frac{e^2}{4}\sqrt{3} + A_2 \quad \text{Seiten tauschen}$$

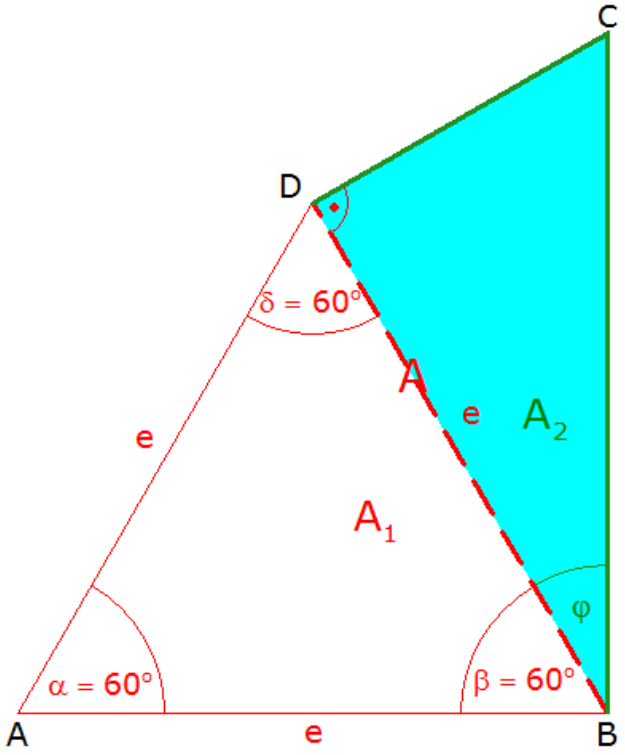
$$\frac{e^2}{4}\sqrt{3} + A_2 = \frac{5e^2}{12}\sqrt{3} \quad \left| -\frac{e^2}{4}\sqrt{3} \right.$$

$$A_2 = \frac{5e^2}{12}\sqrt{3} - \frac{e^2}{4}\sqrt{3}$$

$$A_2 = \frac{5e^2}{12}\sqrt{3} - \frac{3e^2}{12}\sqrt{3}$$

$$A_2 = \frac{2e^2}{12}\sqrt{3}$$

$$\underline{A_2 = \frac{e^2}{6}\sqrt{3}}$$



3. Berechnung der Strecke $\overline{CD}$:

$$A_2 = \frac{\overline{BD} \cdot \overline{CD}}{2}$$

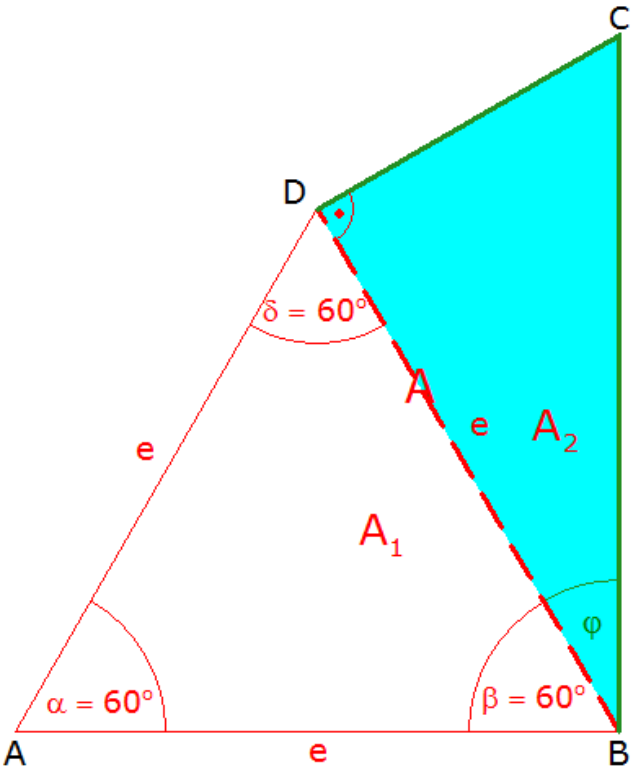
$$\frac{e^2}{6}\sqrt{3} = \frac{e \cdot \overline{CD}}{2} \quad \text{Seiten tauschen}$$

$$\frac{e \cdot \overline{CD}}{2} = \frac{e^2}{6}\sqrt{3} \quad \left| \cdot 2 \right.$$

$$e \cdot \overline{CD} = \frac{e^2}{6}\sqrt{3} \cdot 2$$

$$e \cdot \overline{CD} = \frac{e^2}{3}\sqrt{3} \quad \left| : e \right.$$

$$\underline{\overline{CD} = \frac{e}{3}\sqrt{3}}$$



Lösung 1993 3c:

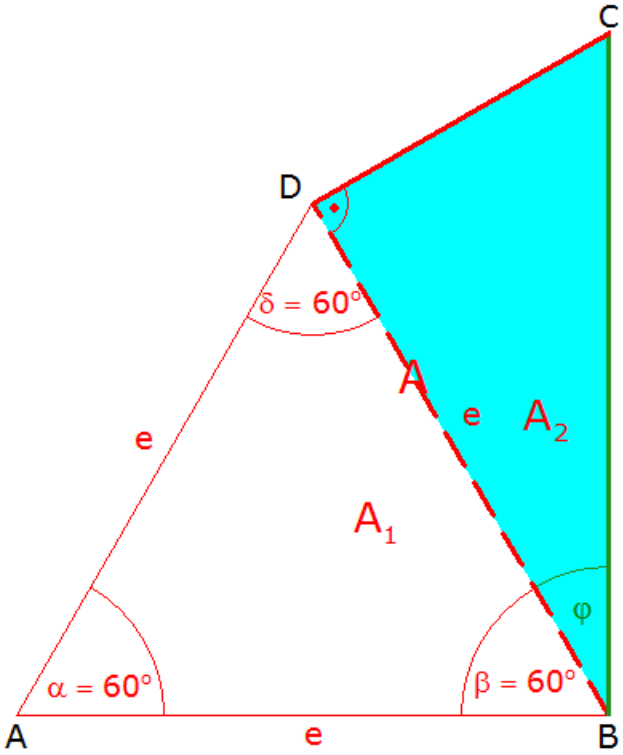
4. Berechnung des Winkels φ:

$$\tan\varphi = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{CD}}{\overline{BD}}$$

Tangensfunktion im rechtwinkligen hellblauen Teildreieck BCD

$$\tan\varphi = \frac{\frac{e}{3}\sqrt{3}}{e}$$

$$\tan\varphi = \frac{e}{3}\sqrt{3} \cdot \frac{1}{e}$$

$$\tan\varphi = \frac{1}{3}\sqrt{3}$$

$$\underline{\underline{\varphi = 30^\circ}}$$